

# Experimental Verification of Computational Models for Laminated Composites

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## Introduction

Structural damage tolerance has been an underlying issue driving much of the research in laminated composite materials. Damage tolerance is broadly defined as the attribute of a structure that permits it to retain its required residual strength for a period of usage after the structure has sustained specific levels of fatigue, corrosion, accidental or discrete source damage. Continuous fiber-reinforced composite materials are generally fatigue and corrosion resistant and are typically designed to be tolerant of nonvisible impact damage and fail-safe from discrete source damage accidents. The design of a damage tolerant structure requires a methodology capable of determining the residual strength of a major structural component after damage. Historically, metallic airframe primary structures have been designed to sustain large crack-like through-penetrations, such as a 2-bay skin crack with a broken frame, that characterize threats such as an uncontained engine failure. Composite primary structures must also be designed for the same threats as metallic structures. Therefore, methodology that can be used to determine the residual strength of a composite structure with through-penetration damage must be developed and verified.

The analysis that is discussed in this paper is based on the damage-dependent constitutive model of Allen and Harris [1,2]. The Allen-Harris model utilizes kinematic-based volume averaged damage variables to represent the effects of matrix cracking and fiber fracture. This model has a matrix crack growth law for fatigue as well as monotonic tension. The kinematic effects of delaminations are modeled empirically.

The objective of the research reported herein is to develop a progressive damage methodology capable of predicting the residual strength of continuous fiber-reinforced, laminated, polymer matrix composites with through-penetration damage. The fracture behavior of center-notch tension panels, see Figure 1, with thin crack-like slits was studied. Since fibers are the major load-carrying constituent in polymer matrix composites, predicting the residual strength of a laminate requires a criterion for fiber fracture. The effects on fiber strain due to other damage mechanisms such as matrix cracking and delaminations must also be modeled. Therefore, the research herein examines the damage mechanisms involved in trans laminate fracture and identifies the toughening mechanisms responsible for damage growth resistance in brittle epoxy matrix systems. The mechanics of matrix cracking and fiber fracture are discussed as is the mathematical framework for the

progressive damage model developed by the authors. The progressive damage analysis algorithms have been implemented into a general purpose finite element code [3] developed by NASA, the **Computational Structural Mechanics Testbed (COMET)**. Damage growth is numerically simulated and the analytical residual strength predictions are compared to experimental results for a variety of notched panel configurations and materials systems.

## Experimental Procedure

The investigation of damage growth resistance in composite laminates is facilitated by the center-notch tension (CNT) specimen, Figure 1. The panel in Figure 1 was manufactured for three panel widths, 10cm, 30cm, and 91cm, and three notch lengths 2.54cm, 7.62cm, and 22.86cm using the material AS4/938 graphite fibers and epoxy matrix. The

laminate stacking sequence is  $[\pm 45/0/90/\pm 30/0]_S$  for all panels. The 10cm and 30cm wide panels were cut from the 91cm wide panel after the fracture test of the 91cm wide panel was complete. The notches were made using a machining process called electronic discharge machining (EDM).

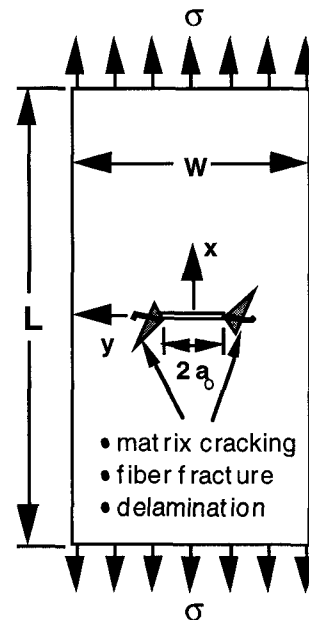


Figure 1. Center-Notch Tension Panel.

The 10cm wide notched panels were monotonically loaded to failure in a 223 kN servo-hydraulic testing machine. The 30cm and 91cm wide panels had anti-buckling guide plates attached just above and below the notch. They were loaded to failure in a 445 kN and 2225 kN servo-hydraulic testing machine, respectively. Specimens were strain gaged and the strain data and applied load from the load cell were recorded. A ring gage was secured in the center of the notch and the center-notch opening displacement from the ring gage was also recorded.

### Quantifying Damage Growth

During the monotonic loading of these panels, the applied load and the crack opening displacement (COD) at the center of the notch was recorded and used to produce load/COD plots, as shown in Figure 2 for the AS4/938 specimen D1AK5A. Discontinuities, or *jumps*, exist at various places along the load/COD plot where the fiber fracture was audible during loading. At these discontinuities, the specimen was unloaded to take an x-ray and are labeled A, B, C, and D on the load/COD plot. It was observed that for each consecutive loading, additional damage did not occur until the loads at which previous damage had occurred were exceeded. The corresponding x-ray radiographs are given in Figure 3 to illustrate the amount of damage at each discontinuity. The x-ray radiographs are actual size.

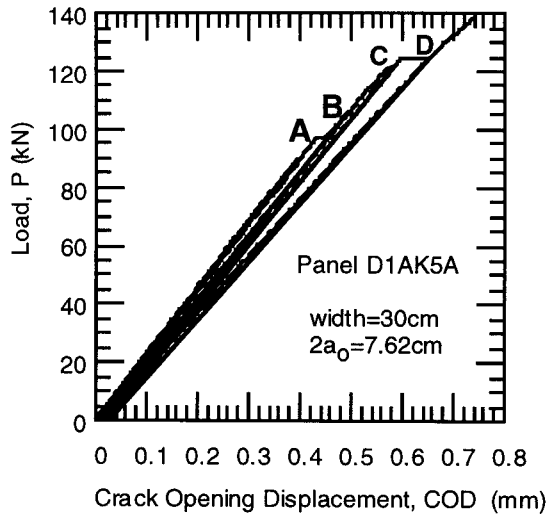


Figure 2. Load/COD Plot for Panel D1AK5A.

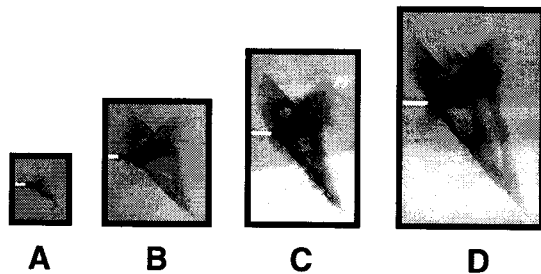


Figure 3. X-Ray Radiographs of Panel D1AK5A Notch-Tip Progressive Damage.

### Progressive Damage Analysis

Observations of damage growth resistance in the experimental study led to an analytical study of R-curves using a progressive damage methodology. This methodology consists of a multi-purpose finite element code [3] and was recently developed to include a residual strength prediction capability [4]. The finite element analysis uses quadrilateral plate or shell elements to analyze general structural geometries. The progressive damage model is damage dependent and can therefore model the damage development at and around the notch-tip for any laminate stacking sequence.

### Nonlinear Damage-Dependent Constitutive Model

The nonlinear, damage-dependent constitutive model of Allen and Harris [1,2] predicts the formation of intraply matrix cracks and fiber fracture for monotonic tensile loading and for tension-tension fatigue [5] (not discussed herein), the associated ply level stress and strain states, and the residual strength of laminates. In order to predict the initiation and growth of delamination, three-dimensional stress states such as those at free edges must be calculated. The analysis currently does not have this capability, therefore, delamination is not modeled in the present analysis.

The constitutive model uses internal state variables (ISV) to represent the average effects of local deformation due to the various modes of microcrack damage. This concept is called continuum damage mechanics. The constitutive model may be written as

$$\sigma_{ijL} = Q_{ijkl} \{ \epsilon_{kl} - \alpha_{kl} \}_L \quad (1)$$

where  $\sigma_{ijL}$  are the locally averaged components of stress,  $Q_{ijkl}$  are the ply-level reduced moduli, and  $\epsilon_{klL}$  are the locally averaged components of strain. The internal state variables,  $\alpha_{klL}$ , represent the local deformation effects of the various modes of damage. When the material is subjected to quasi-static (monotonic) loads, the incremental change of the internal state variable is assumed to be

$$d\alpha_{klL} = \begin{cases} f(\epsilon_{klL}, \beta, \gamma, \psi) & \text{if } \epsilon_{klL} > \epsilon_{klcrit} \\ 0 & \text{if } \epsilon_{klL} < \epsilon_{klcrit} \end{cases} \quad (2)$$

where  $\epsilon_{klcrit}$  is the critical tensile failure strain and  $\beta, \gamma$ , and  $\psi$  are scale factors that describe the load carrying capability of the material after the occurrence of mode I (opening mode) matrix cracking, fiber fracture, and mode II (shear mode) matrix cracking, respectively. The physical interpretation of equation (2) is as follows: As long as the strains in a material element (local volume element or finite element) are less than the critical strains,  $\epsilon_{klcrit}$ , no damage exists and the internal state variables have a zero value. When the strains reach their critical value, the element is damaged and this damage is represented by an internal state variable whose value is proportional to the local strain. The proportionality is dependent on the scale factors  $\beta, \gamma$ , and  $\psi$ . Based on these assumptions, when fiber fracture, mode II matrix cracking, or mode I matrix cracking occur in a ply within an element,

the longitudinal, shear, and transverse stresses for that ply in that element are

$$\sigma_{11} = \gamma S_{cr}^x \quad (3)$$

$$\sigma_{12} = \psi S_{cr}^{xy} \quad (4)$$

$$\sigma_{22} = \beta S_{cr}^y \quad (5)$$

where  $S_{cr}^x$ ,  $S_{cr}^{xy}$ , and  $S_{cr}^y$  are the lamina longitudinal, shear, and transverse critical strengths, respectively. Note that damaged elements are not removed. Rather, the value of the internal state variables in the damaged elements increases proportionately with strain such that equations (3), (4) and (5) are not violated.

### Progressive Damage Methodology

The damage dependent constitutive equations (1) are substituted into the laminate resultant force and moment equations to produce damage dependent laminate resultant force and moment equations. These equations are substituted into the plate equilibrium equations resulting in a set of governing differential equations which can be integrated against variations in the displacement components to produce a weak form of the damage-dependent laminated-plate equilibrium equations [3]. By substituting displacement interpolation functions into the weak form of the plate equilibrium equations and following well known finite element procedures, the assembled equilibrium equations are obtained as

$$[K]\{d\} = \{F_A\} + \{F_D\} \quad (6)$$

where  $[K]$  is the original global stiffness matrix,  $\{d\}$  is the global displacement vector,  $\{F_A\}$  is the applied force vector, and  $\{F_D\}$  is the damage induced force vector. Since the effects of damage are represented as damage-induced force vectors on the right hand side of equation (6), the element stiffness matrix need not be recalculated as damage progresses as long as the nonlinearity in the load-deflection curve is not large.

### Experimental and Computational Results

Figure 4 presents the extent of damage predicted by the analytical model for the 91 cm wide AS4/938 panel. A dimensioned drawing of the center-notch panel is shown at the top left with a small box drawn around the notch-tip region that represents the approximate area covered by the mesh displayed to the right of the drawing. The mesh uses shaded elements, providing a damage contour, to represent the modeled damage at 88% of the predicted ultimate failure load. The damage contour in Figure 4(a) illustrates the ply-level fiber fracture in the elements near the notch-tip. Mode I and mode II matrix cracking are illustrated in Figures 4(b) and 4(c), respectively. An interesting observation of Figure 4(a) is that the  $-45^\circ$  ply exhibits much less progression of fiber fracture than the  $+45^\circ$  ply. This is a numerical result of using a strain failure criterion to initiate damage. If a uniaxial tensile load is applied to an unnotched composite laminate,  $\epsilon_1$  equals  $\epsilon_2$  for the  $+45^\circ$  and  $-45^\circ$  plies. If the laminate has a notch, then the resultant laminate shear force is no longer equal to zero, and  $\epsilon_1$  does not equal  $\epsilon_2$  within a  $45^\circ$  ply.

Instead,  $\epsilon_1$  of the  $-45^\circ$  ply is equal to  $\epsilon_2$  of the  $+45^\circ$  ply, and  $\epsilon_2$  of the  $-45^\circ$  ply is equal to  $\epsilon_1$  of the  $+45^\circ$  ply. The result is as shown in Figure 4(a), fewer fibers in the  $-45^\circ$  ply failed because  $\epsilon_1$  in the  $-45^\circ$  ply barely exceeded the critical strain. These strains did eventually become critical as the applied load approached the load at which equilibrium was unattainable (previously defined as the catastrophic failure load).

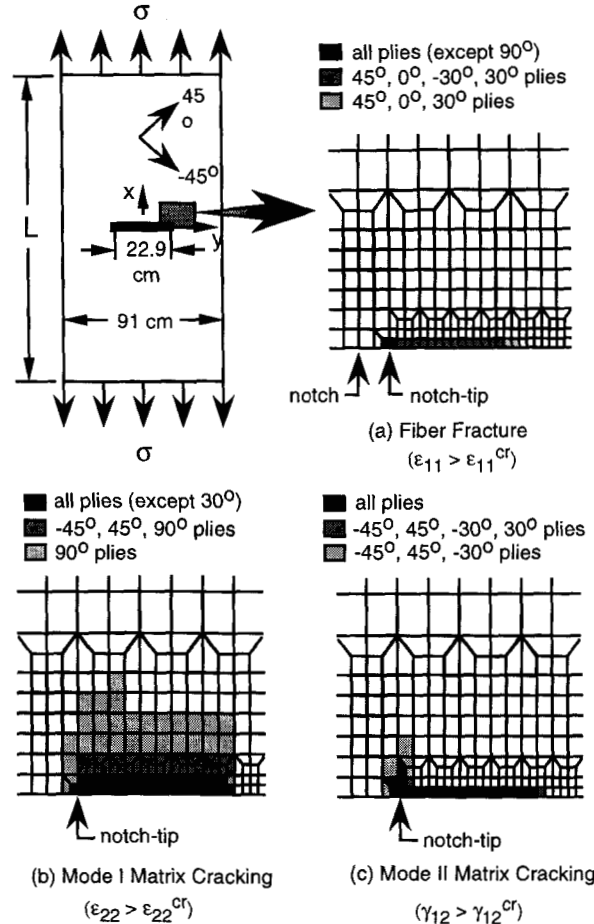


Figure 4. Damaged Elements Displayed for the AS4/938 Panel at 88% of the Ultimate Failure Load. Width = 91 cm,  $2a = 22.9$  cm.

These figures were drawn for an equilibrated solution and illustrate the capability in the analysis to model stable damage growth. Fiber fracture has extended nominally 2.7 cm from the notch-tip. Mode I and mode II matrix cracking has progressed to 3.2 cm and 3.0 cm, respectively. Excluding delamination, the stable damage growth illustrated in these figures is qualitatively similar to what was observed in the experiments. There are elements with fractured plies adjacent to intact plies and mode I matrix cracking occurs in many more elements than the other modes of damage.

The fracture of the center-notch geometry exhibits a panel width and notch size effect. This behavior is best observed by the plot of residual strength versus notch size shown in Figure 5. The experimental average values and the model predictions are compared to the classical linear elastic fracture mechanics (LEFM) results for the panel made of the

AS4/938 material system. According to LEFM, for the same ( $2a_0/W$ ) ratio, the ultimate strength is inversely proportional to the square root of the half notch length. This data is plotted in Figure 5 as the solid line labeled LEFM and was obtained by determining the fracture toughness of the laminate from the residual strength of the smallest specimen with a ( $2a_0/W$ ) ratio equal to 1/4. Notice that the experimental values for the 22.9 cm notch are about 45% higher than the LEFM based predictions. The progressive damage model correctly predicts this increase in the fracture strength. From these results, it is obvious that the finite element model predictions are more accurate than LEFM for wide panels where the fracture resistance effects are dominant. This size effect is produced by toughening mechanisms exhibited by the laminate and is correctly predicted by the progressive damage model.

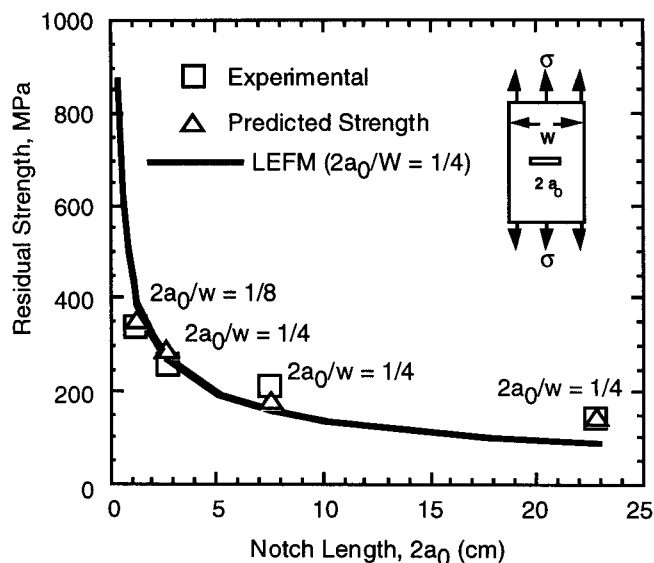


Figure 5. Comparison of LEFM and Model Prediction for the AS4/938 Center-Notch Tension Panels.

### Concluding Remarks

A progressive damage methodology has been developed to predict damage growth and residual strength of fiber-reinforced composite structure with through penetrations such as a slit. The methodology consists of a damage-dependent constitutive relationship based on continuum damage mechanics. Damage is modeled using volume averaged strain-like quantities known as internal state variables. The progressive damage analysis algorithms have been implemented into a general purpose finite element code developed by NASA, the Computational Structural Mechanics Testbed (COMET). Strain failure criteria are used to initiate damage and the internal state variables are used to numerically simulate damage growth. Damage is represented in the equilibrium equations as damage induced force vectors instead of the usual degradation and modification of the global stiffness matrix. Load is applied incrementally and a classical iterative method is used to establish equilibrium.

The progressive damage methodology was used to model the damage at and around the notch-tip. This analysis accounted for mode I and II matrix cracking and fiber

fracture only. Analytical predictions of damage growth compare qualitatively to damage seen in x-ray radiographs of the tested panels. Analytical predictions of residual strength were made for composite panels with central through slits and the computational results agree within  $\pm 10\%$  of the experimental results.

### References

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